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Effect of enhanced sublimation from an active region of a cometary nucleus

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Preliminary results of calculation of the effect of amplification of sublimation due to concentration of the solar radiation by a conical hole in the surface dust layer on a cometary nucleus are given. The temperature of ice on the bottom of the crater is calculated for different geometrical parameters of the crater. The dependence of the effect of intensification of sublimation on the geometrical parameters is considered for a specific case when a single active region is exactly on the pole of the nucleus and this pole is directed to the Sun.

ЭФФЕКТ УСИЛЕНИЯ СУБЛИМАЦИИ ИЗ АКТИВНЫХ ОБЛАСТЕЙ КОМЕТНОГО ЯДРА, Иванова А.В., Шульман Л.М. — Рассматриваются предварительные результаты расчета эффекта усиления сублимации путем концентрации солнечного излучения коническим отверстием в пылевом слое кометного ядра. Рассчитана температура льда на дне кратера в зависимости от его геометрических параметров и эффект усиления сублимации. Рассмотрена зависимость усиления сублимации от геометрических параметров для частного случая, когда единственная активная область находится точно на полюсе ядра и полюс обращен к Солнцу.

ЕФЕКТ ПІДСИЛЕННЯ СУБЛІМАЦІЇ З АКТИВНИХ ОБЛАСТЕЙ КОМЕТНОГО ЯДРА, Іванова О.В., Шульман Л.М. — Розглядаються попередні результати розрахунку ефекту підсилення сублімації криги шляхом концентрації сонячного випромінювання конічним отвором в пиловому шарі кометного ядра. Розраховано температуру на дні кратера в залежності від його геометричних параметрів, та ефект підсилення сублімації.

1. INTRODUCTION

We know that cometary nuclei are non-spherical and their surfaces are inhomogeneous. It is necessary to replace the classic idealized models of a cometary nuclei by more realistic ones.The classic icy model of a nucleus was developed by Whipple [14,15] who considered a cometary nucleus as a conglomerate of frozen gases and dust particles. This model was improved many times [12] afterward. Nuclei of the majority of comets are coated by dust layers, this is shown by laboratory simulation [4,6,7], theoretical calculations [10,12,13] and observations. The dust mantle prevents ice of a cometary nucleus from direct heating by solar radiation. Therefore outgassing of a cometary nucleus is often localized in several active areas. It seems that an active region on a nucleus appears after a local break of the dust mantle. The subject of the present paper is to describe the heat transfer and sublimation in an active region which has been created before by some mechanism unspecified here.

2. STRUCTURE SURFACE OF A COMETARY NUCLEUS

An idealized spherically symmetric model of a cometary nucleus does not take into account any macroscopic non-uniformity of their surfaces. Many of cometary nuclei are covered by dust mantles which attenuate both the heat flux to ice and the gas flow outward because both these transfer processes are diffusion through the pores in the dust. As it was firstly shown in [11] the diffusion molecular flux is considerably less than that from the bare ice surface. Therefore practically all the atmosphere of a comet is formed by sublimation from the active areas.

A simple model of an active region that looks like a conic hole (crater) in the dust mantle of a nucleus is proposed. There is bare ice on the bottom of the cone and dust on its side surface. As the first step the simplest case is considered when the active area is exactly on the pole of the cometary nucleus and the axis of its rotation is directed toward the Sun. This case is not unreal. As it became clear after the DS-1 mission Comet Borrelly nucleus had its active regions near its pole. In this simple case there is no effect of axial rotation on the heating and sublimation. Nevertheless such an approach is good enough to check whether the crater structure of an active region can intensify sublimation.

3. HEAT BALANCE FOR OUR MODEL

There are many papers where the temperature of cometary nuclei was calculated theoretically [2,3,6,9,13] and measured experimentally in the laboratory simulation of the cometary nuclei [7]. The temperature of a nucleus depends on the absorbed solar energy, chemical composition and structure of the nucleus. The calculation of the temperature of a cometary nucleus has been carried out firstly by Dobrovolsky [3] and then by his pupil Markovich [9]. If the crater structure is absent then the boundary condition is the equation of heat balance on the surface of the nucleus, i.e. the relationship:

$$\frac{q(1-A)\cos z}{r^2} = \varepsilon \sigma T^4 - (K\nabla T)_{R(\vec{r},t)=0} + ZL, \tag{1}$$

where q is the solar constant, z is the zenith distance of the Sun, A is the albedo of the surface of the nucleus, ε is its emissivity, is the Stefan–Boltzmann constant, Z is the flux of sublimate from unit surface, L is the energy of sublimation per one molecule, $R(r,t)=0$ is the equation of the surface of the nucleus.

Under the taken here restrictions the active region is in heat equilibrium. It means that the temperature does not depend on time, and the conductive flux of energy inward may be neglected. The temperature distribution should be calculated taking into account that solar radiation reaches bare ice (the bottom of the crater) by two ways: directly from the Sun and by the re-emission in infrared from the walls of the crater. Thus ice on the bottom receives more energy per unit area than in the case when the crater is absent.

The heat balance equation should be written for ice bottom and dust walls of crater separately.

4. NUMERICAL ANALYSIS

A discrete grid was constructed for numerical solution of the system of integral equations (1). Both the bottom and the lateral surface of the cone were divided into small elements of equal areas. The geometry of an active area is determined by the following three quantities: the bottom radius r of the crater, its height h and half vertex angle κ. Let the subscript s denotes the temperature of an element of the dusty side surface and the subscript b refers to the icy bottom. One can rewrite the integral equations (1) as a system of the nonlinear algebraic equations. Using the cylindrical symmetry one can reduce this two-dimensional problem to one-dimensional. So it is possible to calculate the distribution of the temperature over the meridian cross section of a conical crater. The algebraic system can be solved by an iteration method.

To use iterations the initial non-linear system should be transformed into a special form where the next approximation of the temperature (the left-hand sides) is being calculated by substitution the previous approximation into the right-hand sides of the equations. As it is known such an iteration process converges if the functions in the right-hand sides of the transformed equations give so-called contracting projection, i.e. each following approximation must reduce errors of the previous one. Process is terminated when the difference between two consequent approximations is less than the necessary accuracy. To calculate the temperature of conical craters we used such an iteration process:

$$T_{si} = \left\{ \frac{1}{\sigma} \left[\frac{q\sin\kappa}{r_{com}^2} + \sum_{j\neq i}\sum \frac{\sigma T_{sj}}{\pi} \frac{\cos\beta_i \cos\beta_j}{l_{ij}^2} ds - \alpha(2k(T_{si}-T_i)) \right] \right\}^{1/4} \tag{2}$$

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Table 1. Geometrical parameters for different model of active areas on the surface of comet

| r, cm	90	150	160	210	220	240	250
R, cm	300	330	340	360	370	450	500
κ, degree	30	34	36	42	47	56	60

for the dusty side surface and

$$
T_{bi} = \cfrac{B}{A - \ln\left\{\cfrac{\sqrt{2\pi mkT_{bi}}}{L}\left[\cfrac{q(1-A_{ice})}{r_{com}^2} - \sigma T_{bi}^4 + \sum\sum \cfrac{\sigma(1-A)}{\pi}\cfrac{\cos\beta_i \cos\beta_j}{l_{ij}^2}ds\right]\right\}} \tag{3}
$$

for ice bottom of crater. It occurred that this iteration process converged quickly. To solve the integral equations of energy balance numerically a discrete computational greed has been built. Both the bottom and side surface of the cone were divided into small elements of the equal area.

5. RESULTS

To analyze the possible effect we calculated the distribution of the temperature over a meridian cross section of the conical craters with different geometrical parameters: r — radius bottom of crater, H — height of crater, κ — half vertex angle of the cone. The set of their geometrical parameters is given in
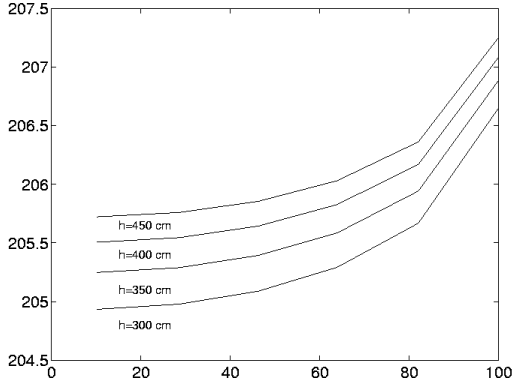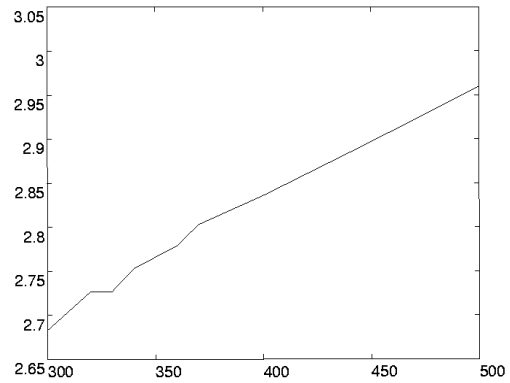

Fig. 1. The temperature distribution $T(r_b)$ for different depth of the cone
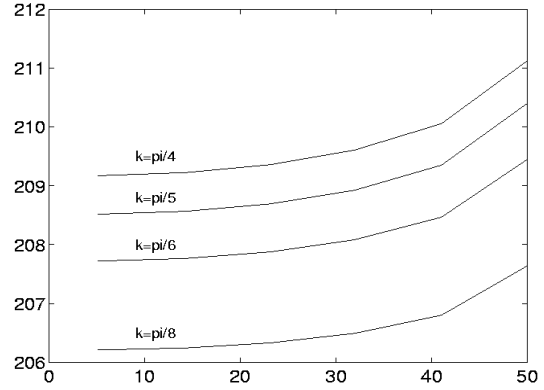
Fig. 2. The temperature distribution $T(r_b)$ for different vertex angle of the cone
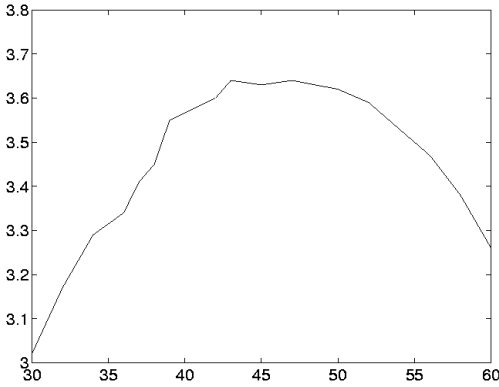
Fig. 3. Amplification of sublimation as a function of the vertex angle of the cone

Fig. 4. Amplification of sublimation as a function of the depth of the cone

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the Table 1. Due to axial symmetry it was enough to calculate the temperature distribution only over a meridian cross-sections of a crater. Then the total flux of water vapour was calculated. On the next step we calculated the relation of this flux to that calculated without additional heating by the wall infrared radiation (the coefficient of amplification of sublimation). Then the flux of sublimate was calculated with the Knudsen–Hertz formula:

$$Z = \frac{p_n(T)}{\sqrt{2\pi mkT}} \qquad (4)$$

where $p_n(T)$ is the pressure of saturated vapour at the given temperature of ice. The total flux of water from the bottom of a crater is given by the relationship:

$$Q = 2\pi \int\limits_0^r \frac{re^{A-B/T(r)}}{\sqrt{2\pi mkT(r)}} \, dr \approx n_b \cdot \Delta S \cdot \sum_{i=1}^{n_b} \frac{e^{A-B/T_i(r)}}{\sqrt{2\pi mkT_i(r)}} \qquad (5)$$

The values $A = 31.085$, $B = 6120.3K$ were taken for the approximation of the saturation pressure of water vapour. The sum in the right-hand side of the (5) is taken over all the bottom elements of the cone. If the crater is absent then the gas productivity of a part of the nucleus with the area πr^2 should be

$$Q_0 = \pi r^2 \frac{e^{A-B/T}}{\sqrt{2\pi mkT}}$$

Here the temperature should be calculated for the given heliocentric distance with the (3) where the double sum in the right-hand side (the additional flux of energy from the walls) is omitted. The amplification factor G of the sublimation rate is

$$G = \frac{Q}{Q_0}$$

This quantity has been calculated for each model and the results are shown in Figs. 3, 4, 5 for the different craters. The dependence of the Q factor on the heliocentric distance of the comet is shown on Fig.6.
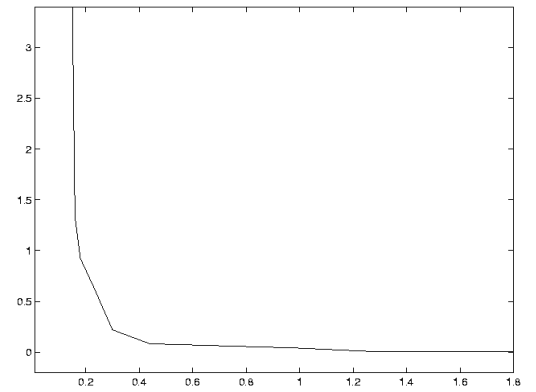
6. GENERALIZATION

Now we consider the case, when the active zone can be located in various parts of rotating cometary nucleus. In such case expression for $\cos(z)$ in mathematical equation (1) not equal 1 and can be se as:

$$\cos(z) = \sqrt{A_1^2 + A_2^2} \cdot \cos(\Phi) \cdot \sin(l - \Lambda + A_3) + A_4 \sin(\Phi),$$
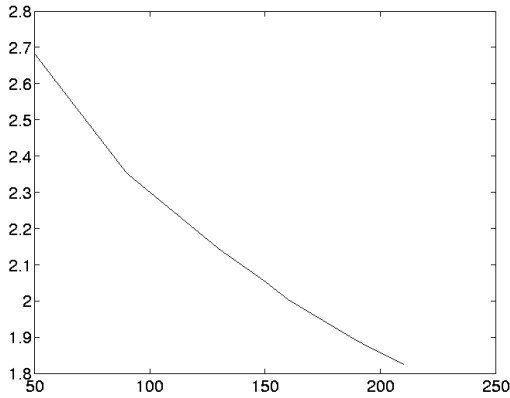
Fig. 5. Amplification of sublimation as a function of the bottom radius of the cone

Fig. 6. Amplification of sublimation as a function of the heliocentric distance of the comet

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where Φ, Λ is nucleocentric latitude and longitude, parameters A_1, A_2, A_3, A_4 can be set as:

$$A_1 = \cos\kappa\cdot\cos^2\beta\cdot\cos\alpha\cdot\cos(\phi_\text{к}-\phi_\text{з})+\cos\kappa\cdot\sin^2\beta\cdot\sin(\phi_\text{к}-\phi_\text{з})\cdot\cos\varepsilon+\sin\kappa\cdot\sin\alpha\cdot\cos\beta\cdot\sin\varepsilon$$

$$A_2 = \cos\kappa\cdot\cos\beta\cdot\sin\beta\cdot\cos\alpha\cdot\cos(\phi_\text{к}-\phi_\text{з})-\cos\kappa\cdot\sin\beta\cdot\cos\beta\cdot\sin(\phi_\text{к}-\phi_\text{з})\cdot\cos\varepsilon+$$
$$+\sin\kappa\cdot\sin\alpha\cdot\cos\beta\cdot\sin\varepsilon$$

$$A_3 = \arctan\frac{A_1}{A_2}$$

$$A_4 = \cos\kappa\cdot\cos\beta\cdot\sin\alpha\cdot\cos(\phi_\text{к}-\phi_\text{з})+\sin\kappa\cdot\cos\alpha\cdot\sin(\phi_\text{к}-\phi_\text{з})\cdot\sin\varepsilon$$

Considering value for $\cos(z)$ we obtain new expression for equation of heat balance. We analyzing obtained temperature and study the affect of nucleus rotation and active zone location on temperature curves.

7. CONCLUSIONS

The obtained results show that a crater structure can intensify sublimation by a factor $\leqslant 3$. It is easy to understand that in more general case (i.e. another localisation of the active area) the cometary nucleus will generate gas-and-dust jets when the active areas are turning on the day side of the nucleus. One can suppose that large dust particles are accumulated on the bottom of the crater forming a mantle there. One can suppose that large dust particles are accumulated on the bottom of the crater forming a mantle there. It is interesting that the temperature of ice on the bottom is increasing from center to the edge (Figs. 1, 2).

Therefore, an initially plain bottom of the crater becomes convex and there should be a possibility to throw out a massive icy fragment. By this reason a spontaneous outburst of sublimation can occur when the dust mantle on the bottom of the crater will be blown up.

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