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Regional gravity field determination from satellite altimetry data in the Black Sea and Azov Sea area

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Gravity anomalies in the Black sea area were determined using satellite altimetry observations of the Sea Surface Heights over this region. The satellite data applied in the analysis include ERS-1, ERS-2, and TOPEX/POSEIDON altimetry from 1992 to 2001. The solutions for gravity anomalies at $(3' \times 3')$ grid points are evaluated by the Tikhonov regularization method to provide estimations based on kernel functions, which are described by singular point harmonic functions. Comparison with independent solution KMS01 of the Danish geodetic institute was performed.

ВИЗНАЧЕННЯ РЕГІОНАЛЬНОГО ГРАВІТАЦІЙНОГО ПОЛЯ ЗА ДАНИМИ СУПУТНИКОВОЇ АЛЬТИМЕТРІЇ В РЕГІОНІ ЧОРНОГО ТА АЗОВСЬКОГО МОРІВ, Марченко О.М., Тартачинська З.Р., Якимович О.М. — Розглянута задача визначення висот геоїда і аномалій сили ваги в регіоні Чорного моря з використанням даних супутникових альтиметричних місій ERS-1, ERS-2 і TOPEX/POSEIDON за період з 1992 р. по 2001 р. Методом регуляризації Тіхонова побудовано поле висот геоїда і аномалій сили ваги з роздільною здатністю $(3' \times 3')$. Виконано порівняння з незалежним розв'язком KMS01 Датського геодезичного інституту.

ОПРЕДЕЛЕНИЕ РЕГИОНАЛЬНОГО ГРАВИТАЦИОННОГО ПОЛЯ ПО ДАННЫМ СПУТНИКОВОЙ АЛЬТИМЕТРИИ В РЕГИОНЕ ЧЕРНОГО И АЗОВСКОГО МОРЕЙ, Марченко А.Н., Тартачинская З.Р., Якимович А.Н. — Рассмотрена задача определения аномалий силы тяжести в регионе Черного моря, используя данные спутниковых альтиметрических миссий ERS-1, ERS-2 и TOPEX/POSEIDON за период с 1992 г. по 2001 г. Методом регуляризации Тихонова построено поле высот геоида и аномалий силы тяжести с разрешением $(3' \times 3')$. Выполнено сравнение с независимым решением KMS01 Датского геодезического института.

1. INTRODUCTION

Recent altimetry satellites give more and more accurate determination of the Sea Surface Heights (SSH), which in combination with an ocean circulation model can be considered as a direct determination of the geoid undulations in different ocean areas. Appropriate data sets from various altimetry satellites are collected also for such relatively small internal marine regions of the globe as Black sea and Sea of Azov. Thus, the inversion of SSH data set into the gravity anomalies Δg becomes possible within the mentioned internal marine areas, assuming corrected SSH data as "measured" geoid undulations N (see, for instance, [2, 5]).

This paper represents further development of the recent results from [5] and will focus on the recovery of the gravity anomalies Δg from the combination of ERS-1, ERS-2, and TOPEX/POSEIDON altimetry (SSH) in the Black and Azov sea area by the Tikhonov regularization method.

The following data sets corrected by AVISO for different geophysical and instrumental effects are used (Table 1):

- subset 1 represents 36836 values of SSH taken for the period from October 1992 to June 1996 of the ERS-1 mission;
- subset 2 represents 122973 values of SSH taken for the period from April 1995 to September 2001 of the ERS-2 mission;

Table 1. Distribution of satellite altimetry data in the Black Sea and Sea of Azov area

Satellite	Period	Number of corrected SSH
ERS1	10.04.1994 – 09.03.1995	17921
ERS1	24.03.1995 – 02.06.1996	18915
ERS1		Total = 36836
ERS2	24.04.1995 – 06.06.1996	16564
ERS2	06.06.1996 – 01.09.1997	19207
ERS2	01.09.1997 – 26.10.1998	25471
ERS2	26.10.1998 – 11.10.1999	22076
ERS2	11.10.1999 – 25.09.2000	20863
ERS2	25.09.2000 – 10.09.2001	18792
ERS2		Total = 122973
TOPEX/POSEIDON	02.10.1992 – 04.10.1993	18649
TOPEX/POSEIDON	04.10.1993 – 16.10.1994	18485
TOPEX/POSEIDON	16.10.1994 – 08.10.1995	19128
TOPEX/POSEIDON	09.10.1995 – 09.10.1996	18076
TOPEX/POSEIDON	09.10.1996 – 11.10.1997	18183
TOPEX/POSEIDON	11.10.1997 – 13.10.1998	17906
TOPEX/POSEIDON	13.10.1998 – 05.10.1999	17310
TOPEX/POSEIDON	06.10.1999 – 06.10.2000	18460
TOPEX/POSEIDON	07.10.2000 – 08.10.2001	15050
TOPEX/POSEIDON		Total = 161247

- subset 3 represents 161247 values of SSH taken for the period from October 1992 to October 2001 of the TOPEX/POSEIDON mission.

2. METHOD

As before [5] the traditional "remove–restore" procedure was used to get the initial information δN for further determination of the gravity anomalies Δg:

$$\delta N = SSH - N_{\mathrm{EGM96}}, \tag{1}$$

where SSH are the corrected Sea Surface Heights, assumed to be coincided with the geoid height N; N_{EGM96} is the long wavelength part of N adopted according to EGM96 gravity field model (360, 360).

Then the prediction of the residual gravity anomalies $\delta\Delta g_P$ and the residual geoid heights δN_P was estimated at some point P (inside the studying area) applying the regularization method

$$\delta\Delta g_P = \mathbf{C}_{\delta\Delta g,\delta N}(\mathbf{C}+\alpha\mathbf{C}_{nn})^{-1}\mathbf{l}, \tag{2}$$

$$\delta N_P = \mathbf{C}_{\delta N,\delta N}(\mathbf{C}+\alpha\mathbf{C}_{nn})^{-1}\mathbf{l}, \tag{3}$$

where $\mathbf{l}$ is the q-vector consisting in this case of the components δN_i $(i = 1, 2, \ldots q)$; q is the number of the observations δN_i; $\mathbf{C}$ is the $(q \times q)$-covariance matrix of the residual geoid height δN; $\mathbf{C}_{\delta\Delta g,\delta N}$ is the $(1 \times q)$-cross-covariance matrix between $\delta\Delta g$ and δN; $\mathbf{C}_{\delta N,\delta N}$ is the $(1 \times q)$-auto-covariance matrix of δN; $\mathbf{C}_{nn}$ is the $(q \times q)$-covariance matrix of the measurements noise n; α is the Tikhonov regularization parameter [5–7].

Having the values (1) at some set of scattered points and the above covariance matrixes, the residual gravity anomalies $\delta\Delta g$ and the residual geoid heights δN are estimated straightforward at chosen grid points by the regularization method. After solving this basic problem the predicted gravity anomalies Δg and geoid undulations N were restored at the same grid by means of EGM96 gravity field model

$$\Delta g = \Delta g_{\mathrm{EGM96}} + \delta\Delta g, \tag{4}$$

$$N = N_{\mathrm{EGM96}} + \delta N, \tag{5}$$

For further use of the relationships (2), (3) the following problems have to be solved:

1. The construction of the analytical covariance function $K(P,Q)$ of the anomalous potential T.

2. The choice of a suitable method for the computation of the regularization parameter α

The analytical covariance function or reproducing kernel $K(P,Q)$, described only by singular point harmonic functions [3, 4], is chosen in the following way

$$K_n(P,Q) = \left[\frac{GM}{R}\right]^2 \beta_n \sigma^{n+1} \hat{v}_n, \qquad \sigma = \frac{R_B^2}{r_P r_Q}, \tag{6}$$

where R is the Earth‘s mean radius; R_B is the Bjerhammar’s sphere radius; r_P and r_Q are the geocentric distances to the external points P and Q; GM is the product of the gravitational constant G and the

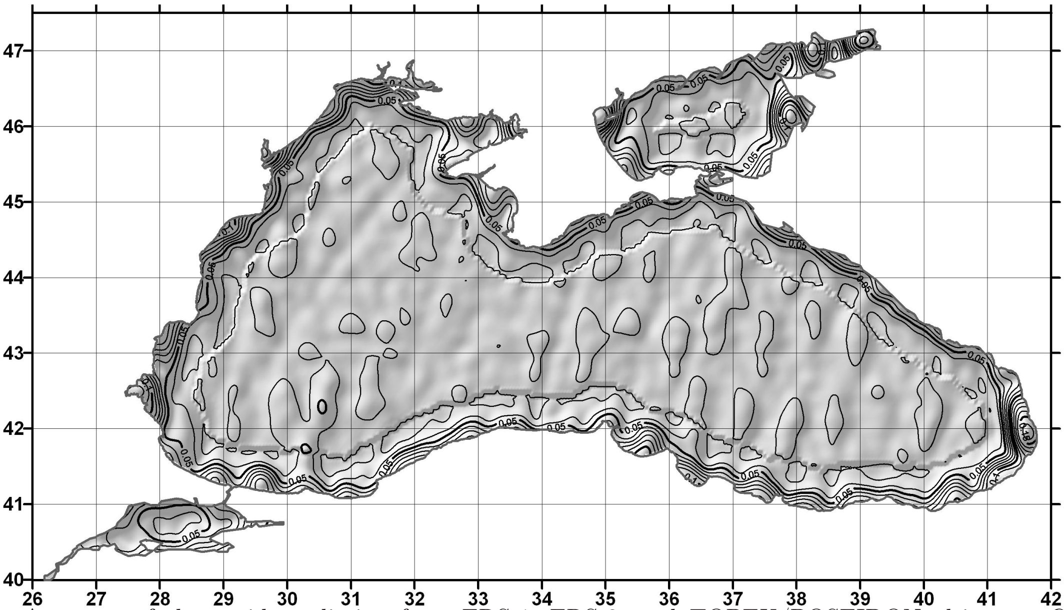

Fig. 1. Accuracy of the geoid prediction from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry. Contour interval: 0.01 m

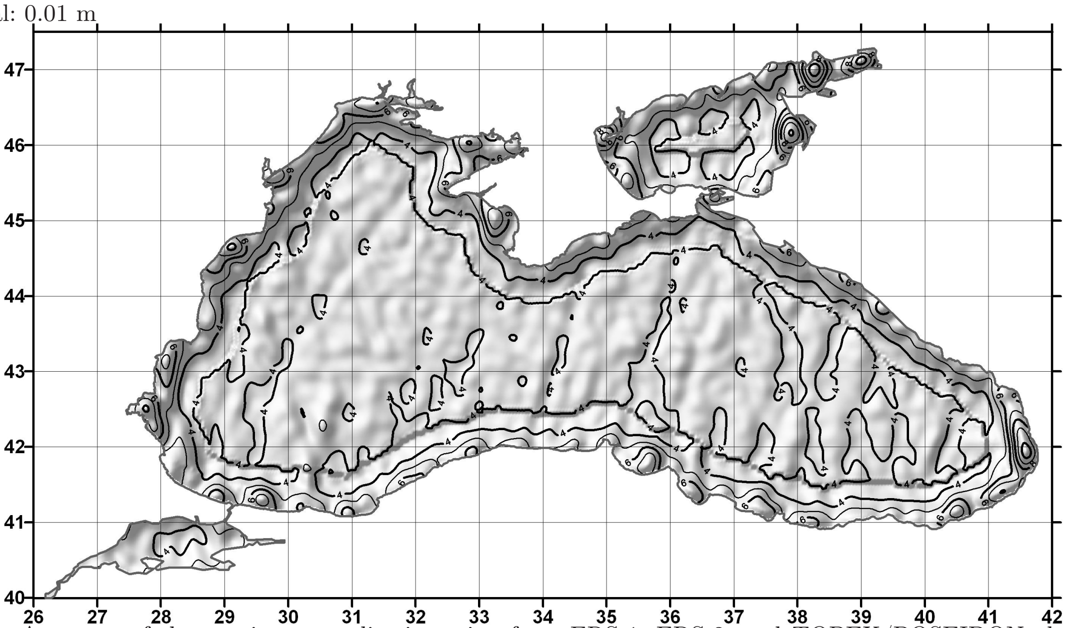

Fig. 2. Accuracy of the gravity anomalies inversion from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry. Contour interval: 1 mGal

planet's mass M; $\hat{v}_n$ is the dimensionless potential of radial multipole of the degree n; β_n represents some dimensionless coefficient. Expressions for the analytical auto-covariance function of geoid heights and cross-covariance function between gravity anomalies and geoid undulations (based on the covariance propagation) can be found in [3].

Note now that the traditional determination of the regularization parameter α in (2) or (3) according to [7, 8] requires in the frame of a special iterative process the inversion of matrixes with a dimension

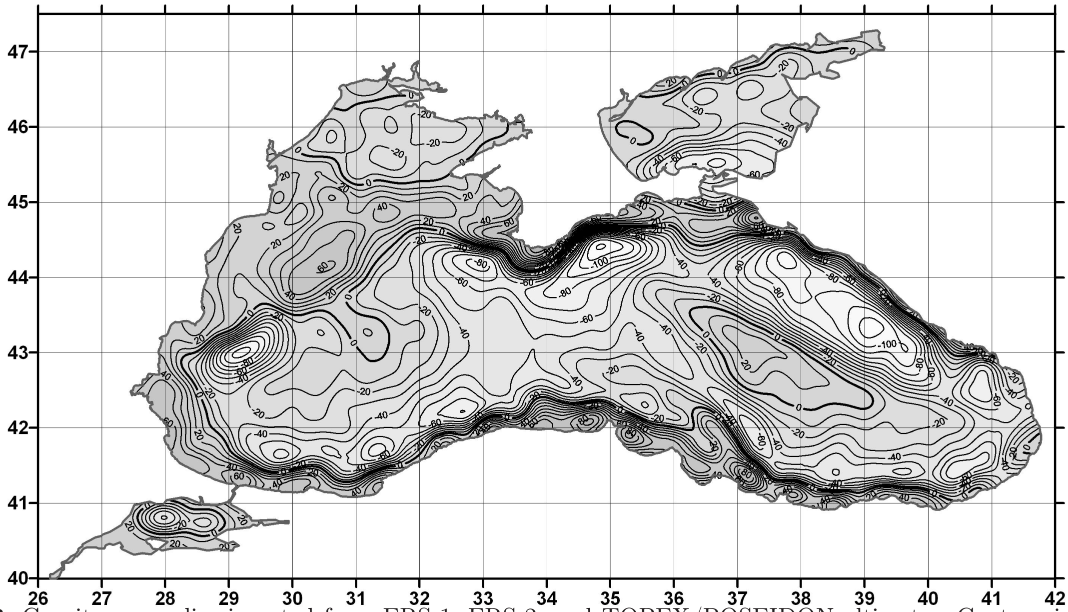

Fig. 3. Gravity anomalies inverted from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry. Contour interval: 10 mGal

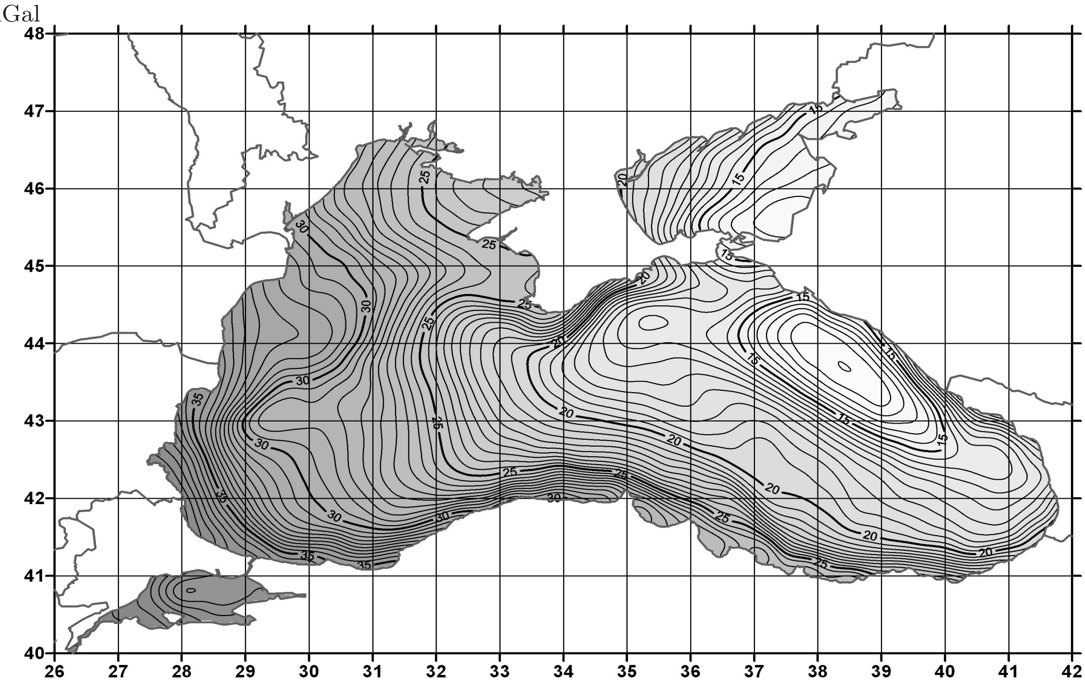

Fig. 4. Geoid heights from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry.

Table 2. Statistics of the predicted residual geoid heights δN and gravity anomalies $\delta\Delta g$ at grid points $(3' \times 3')$

Statistics	N, m	$\delta\Delta g$, mGal
Minimum	-2.48	-81.72
Maximum	1.92	78.29
Mean	-0.57	-7.42
Standard deviation	0.56	15.57

Table 3. Statistics of the geoid heights and gravity anomalies restored at grid points $(3' \times 3')$ and their accuracy estimation

Statistics	N, m	σN, m	Δg, mGal	$\sigma\Delta g$, mGal
Minimum	11.99	0.02	-121.26	3.16
Maximum	40.13	0.17	102.59	10.56
Mean	23.13	0.04	-17.52	4.48
Standard deviation	6.63		37.57	

Table 4. Comparison of the predicted $(3' \times 3')$ geoid heights and $(3' \times 3')$ KMS2001 SSH

Statistic	$N - N_{\mathrm{KMS}}$, m
Minimum	-2.31
Maximum	1.49
Mean	-0.37
St. deviation	0.51

equal to the number q of observations. So, when a number of observations are large we come to a time consuming procedure. As before [5] to avoid this difficulty another possible value of α is used

$$\alpha = 1 + \sqrt{1 + \mathrm{Trace}(\mathbf{CC}_{nn})/\mathrm{Trace}(\mathbf{CC}_{nn})} \tag{7}$$

leading to the estimation of α prior to matrix inversion in (2) and (3).

Simplest illustration of a possible values of the regularization parameter α given by (7) can be made under several assumptions. First one, geodetic measurements of one kind only are considered. Second one, the matrix $\mathbf{C}_{nn}$ can be represented as $\mathbf{C}_{nn} = d\mathbf{I}$, where d is the variance of a noise and $\mathbf{I}$ is the unite matrix. Third one, the matrix $\mathbf{C}$ can be described by the Dirac delta function and can be written as $\mathbf{C} = C_0\mathbf{I}$, where C_0 is the variance of a studying field. With these assumptions the expression for the regularization parameter corresponded to (7) can be found as

$$\alpha = 1, \tag{8}$$

$$\alpha = 1 + \sqrt{1 + \frac{C_0}{d}}. \tag{9}$$

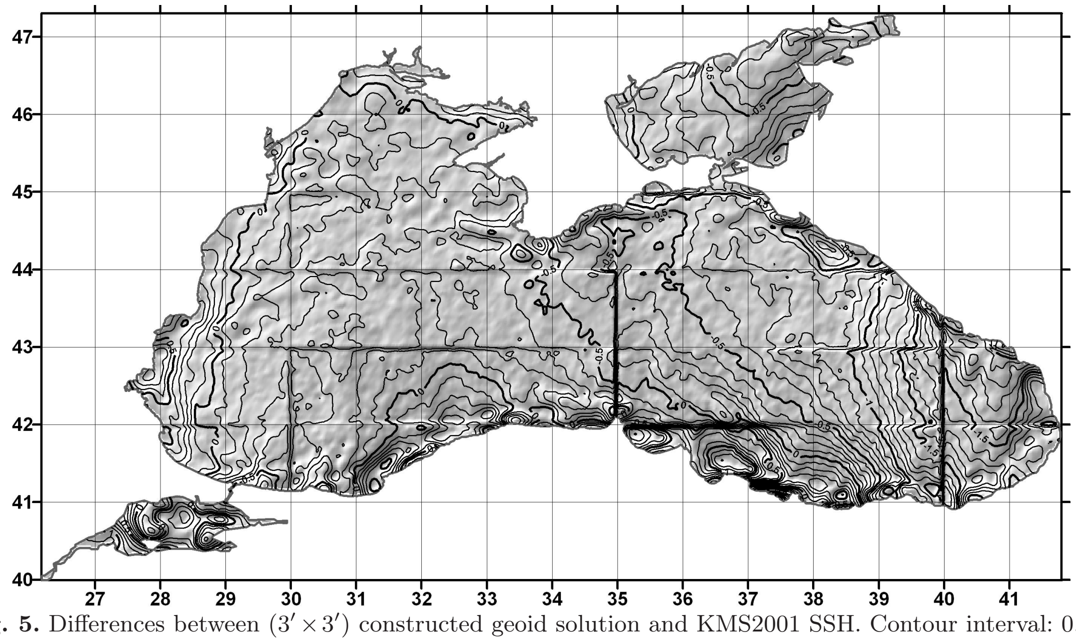

Fig. 5. Differences between $(3' \times 3')$ constructed geoid solution and KMS2001 SSH. Contour interval: 0.1 m

In fact, the first root (8) corresponds in (2) and (3) to the least-squares collocation solution. The second root (9) corresponds to the relationship (7) under the adopted assumptions and can serve for the illustration of a possible dependence of α on the given C_0 and d.

Note again that the formulae (7) and (9) represent only possible upper limit of α [5], which requires a further improvement of the considered estimation of α.

3. RESULTS AND CONCLUSIONS

Removing the contribution of the geopotential model EGM96 (360,360) from altimetry data (SSH) the residual geoid heights δN were adopted as initial information. Then the empirical covariance function (ECF) of the residual geoid heights $\delta N = \delta\, SSH$ was constructed and approximated by the analytical covariance function (ACF) based on the radial multipoles potentials or ACF of the so-called point singularities [4]. As a result, the optimal degree $n = 1$ in the formula (6) was chosen from ECF approximation that corresponds to the dipole kernel function (Poisson kernel without zero degree harmonics). The optimal ACF has the following essential parameters: (a) the variance of field - $\mathrm{var}(N) = 0.259\, m^2$; (b) the correlation length $\xi = 0^\circ.449$; (c) the curvature parameter $\chi = 4.088$. Note that this optimal ACF has the same parameters as constructed by [5] in the preliminary solution despite of essentially larger number of observations SSH.

According to the expressions (2), (3) and (7), the prediction of the residual gravity anomalies $\delta\Delta g$ and the residual geoid heights δN was done by the regularization method at the adopted 20770 grid points with the resolution $(3' \times 3')$. Statistics of the estimated δN and $\delta\Delta g$ and their accuracy are shown in the Table 2.

Accuracy distributions are shown in Fig. 1, 2, 3, 4 illustrate the geoid heights and gravity anomalies, respectively, based on the 321056 SSH from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry and predicted by the regularization method.

Table 4 and Fig. 5 illustrates the comparison of the constructed above geoid solution from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry and $(3' \times 3')$ KMS2001 SSH derived from ERS-1, GEOSAT and TOPEX/POSEIDON data. Fig. 5 reflects also the adopted in KMS approach [1] of the "piecewise processing" within every $1^\circ \times 5^\circ$ chosen rectangular cell because data processing in this study was made for the whole geographical region without any separation to cells.

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